SAARLAND UNIVERSITY
Department of Mathematics
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Mathematics for Computer Scientists 1, WS 2017/18
Sheet 3

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1. The following tables show the results of the arithmetical operations in $\mathbb{Z}_3$ (where $\oplus$ and $\odot$ denote addition and multiplication modulo 3).

$\oplus$	$[0]$	$[1]$	$[2]$
$[0]$	$[0]$	$[1]$	$[2]$
$[1]$	$[1]$	$[2]$	$[0]$
$[2]$	$[2]$	$[0]$	$[1]$

$\odot$	$[0]$	$[1]$	$[2]$
$[0]$	$[0]$	$[0]$	$[0]$
$[1]$	$[0]$	$[1]$	$[2]$
$[2]$	$[0]$	$[2]$	$[1]$

(a) Compute the corresponding tables for $\mathbb{Z}_5$ and $\mathbb{Z}_7$.

(b) Compute the corresponding tables for $\mathbb{Z}_4$ and show that $(\mathbb{Z}_4, \oplus, \odot)$ is not a field.

2. Let $n \geq 2$ be a natural number and $(\mathbb{Z}_n, \oplus, \odot)$ be a field (where $\oplus$ and $\odot$ denote addition and multiplication modulo n). Show that n is a prime number.

[Hint: Suppose that n is not a prime number, so that there exist $a, b \in \{2, \ldots, n-1\}$ with $n = ab$. What is $[a] \odot [b]$?]

3. Define the binary operations 'subtraction' and 'division' on the set of real numbers. Let a, b, c, d be real numbers with b, $d \neq 0$. Show that

$$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}, \qquad \frac{a}{b} \bigg/ \frac{d}{c} = \frac{ac}{bd},$$

using *only the axioms of arithmetic and your definitions.*

4. Determine the real numbers x for which the following inequalities hold.

(a) $\dfrac{4x-5}{x^2-1} < 5$

(b) $\dfrac{5}{5x-1} < \dfrac{2}{2x+1}$

(c) $\dfrac{3x+2}{2x+3} < \dfrac{x}{x+1}$

(d) $\left|\dfrac{(x-1)(2x-3)}{x(x-5)}\right| > 1$

(e) $\log\left(\dfrac{2-x}{12+4x}\right) > 0$

(f) $\mathrm{e}^x > 3^{x^2}$

5. Sketch the subsets

$$\begin{aligned} A_1 &= \Big\{(x,y) : 3x+2y \le 6,\ x-y \le 2,\ x \le 1\Big\}, \\ A_2 &= \left\{(x,y) : |y| \le \frac{\sqrt{5}}{2},\ |y-\sqrt{5}x| \le \sqrt{5},\ |y+\sqrt{5}x| \le \sqrt{5}\right\} \end{aligned}$$

of the (x,y) coordinate plane.