SAARLAND UNIVERSITY
Department of Mathematics
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Mathematics for Computer Scientists 1, WS 2018/19
Sheet 10

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1. (a) Find all convergent subsequences of the sequence

$$1, -1, -1, 1, 1, 1, -1, -1, -1, -1, 1, 1, 1, 1, 1, \ldots.$$

(b) Find all convergent subsequences of the sequence

$$1, 2, 2, 1, 2, 2, 3, 3, 3, 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, \ldots.$$

(c) For which real numbers α is there a subsequence of the sequence

$$\tfrac{1}{2}, \tfrac{1}{3}, \tfrac{2}{3}, \tfrac{1}{4}, \tfrac{2}{4}, \tfrac{3}{4}, \tfrac{1}{5}, \tfrac{2}{5}, \tfrac{3}{5}, \tfrac{4}{5}, \ldots,$$

which converges to α?

2. (a) Derive the formula

$$\sum_{k=1}^{n} \frac{1}{k(k+1)(k+2)} = \frac{1}{4} - \frac{1}{2(n+1)} + \frac{1}{2(n+2)}$$

[Hint:

$$\frac{1}{k(k+1)(k+2)} = \frac{1}{2k} - \frac{1}{k+1} + \frac{1}{2(k+2)}$$

for all $k \in \mathbb{N}$.]

(b) Prove that the series

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$$

converges and determine its sum.

3. Which of the following series are convergent?

(a) $\displaystyle\sum_{r=1}^{\infty} \frac{r^3+4r+3}{\sqrt{r^{10}+r^7}}$

(b) $\displaystyle\sum_{r=1}^{\infty} \frac{r^3+4r+3}{\sqrt{r^8+3r^7}}$

(c) $\displaystyle\sum_{r=1}^{\infty} \frac{1}{(1+1/r)^r}$

(d) $\displaystyle\sum_{r=1}^{\infty} \frac{r^4+1}{2^r}$
[Hint: $r^4+1 \leq (\frac{3}{2})^r$ for large r]

(e) $\displaystyle\sum_{r=1}^{\infty} \frac{r+2^r}{r2^r}$
[Hint: $r+2^r \geq 2^r$]

(f) $\displaystyle\sum_{r=1}^{\infty} \frac{r!}{r^r}$
[Hint: ratio test]

(g) $\displaystyle\sum_{r=1}^{\infty} \sin\frac{1}{r}$
[Hint: $\sin x \geq \frac{1}{2}x$ for small x]

(h) $\displaystyle\sum_{r=1}^{\infty} \frac{1}{r}\sin\frac{1}{r}$
[Hint: $\sin x \leq x$]

(i) $\displaystyle\sum_{r=2}^{\infty} \frac{1}{r\log r}$
[Hint: $\dfrac{\mathrm{d}}{\mathrm{d}x}(\log(\log x)) = \dfrac{1}{x\log x}$]

(j) $\displaystyle\sum_{r=2}^{\infty} \frac{1}{r^2\log r}$
[Hint: $\log r > 1$ for large r]