SAARLAND UNIVERSITY
Department of Mathematics
Prof. Dr. Mark Groves
MSc Jens Horn

Mathematics for Computer Scientists 1, WS 2017/18
Sheet 4

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1. Prove the following assertions by mathematical induction.

(a) $\sum_{i=1}^{n} \log\left(1+\frac{1}{i}\right) = \log(1+n)$ for each natural number n;

(b) $\prod_{i=1}^{n}(2i-1) = \frac{(2n)!}{2^n n!}$ for each natural number n;

(c) $n^2 \leq 2^n \leq n!$ for each natural number $n \geq 4$;

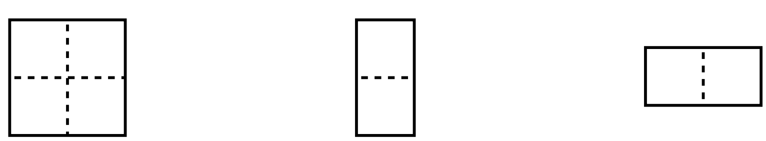
(d) $6|(2^n + 3^n - 5^n)$ for each natural number n.

2. (a) Let $\mathcal{P}(n)$, $n \in \mathbb{N}$ be a predicate with the following properties:

- There exists $m \in \mathbb{N}$ such that $\mathcal{P}(1)$, ..., $\mathcal{P}(m)$ are true.
- Let $k > m$. $P(k)$ is true whenever $\mathcal{P}(j)$ is true for all $j < k$.

Deduce from the well-ordering axiom of the natural numbers that $P(n)$ is true for all $n \in \mathbb{N}$. (This is called **strong induction**.)

(b) The aim of 'mini-tetris' is to fill a $2 \times n$ rectangle (completely, and without overlaps) with tiles of the following type:



Let T_n be the number of ways of filling a $2 \times n$ rectangle with these tiles.

Determine T_1 und T_2, find a formula for T_n for $n \geq 3$ as a function of T_{n-1} and T_{n-2}, and prove by strong induction that

$$T_n = \frac{1}{3}[2^{n+1} + (-1)^n].$$

3. Calculate $(1552303, 233927)$ and find integers m and n such that

$$(1552303, 233927) = 1552303m + 233927n.$$

4. Let a and b be natural numbers and $d = (a, b)$.

(a) Show that d is the smallest element of the set

$$\{ma + nb : m, n \in \mathbb{Z}\} \cap \mathbb{N}.$$

[Hint: You do not need the well-ordering axiom of the natural numbers.]

(b) Suppose there are integers m and n such that $ma + nb = 1$. Deduce that $(a, b) = 1$.