SAARLAND UNIVERSITY
Department of Mathematics
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Mathematics for Computer Scientists 2, SS 2018
Sheet 12

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1. Show that

$$S = \left\{ \begin{pmatrix} 1 \\ -1 \\ \mathrm{i} \end{pmatrix}, \begin{pmatrix} \mathrm{i} \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right\}$$

is a basis for $\mathbb{C}^3$. Apply the Gram-Schmidt procedure to S to find an orthonormal basis for $\mathbb{C}^3$.

2. Show that the matrix

$$A = \frac{1}{9} \begin{pmatrix} -7 & 4 & 4 \\ 4 & -1 & 8 \\ 4 & 8 & -1 \end{pmatrix}$$

lies in $\mathrm{SO}(3)$. Determine the axis and angle of the rotation represented by A.

3. Let M be the $n\times$n matrix whose entries are all 1. Determine the rank and signature of the quadratic form $\mathbf{x}^{\mathrm{T}} M \mathbf{x}$.

[Hint: convert M into diagonal form by symmetric elementary operations.]

4. Sketch the conic sections with equations

(i) $6x^2 + 24xy - y^2 - 12x + 26y + 11 = 0$,

(ii) $52x^2 - 72xy + 73y^2 + 40x + 30y - 75 = 0$.